1). What are the first 4 terms of the sequence of partial sums of $\sum_{n=1}^{\infty} n!$ (2 points)

$$\sum_{n=1}^{\infty} n! = 1! + 2! + 3! + \cdots$$

$S_1 = 1! = 1$

$S_2 = 1! + 2! = 3$

$S_3 = 1! + 2! + 3! = 9$

$S_4 = 1! + 2! + 3! + 4! = 33$

Sequence of Partial sums
1, 3, 9, 33

2) For the following convergent series, (6 points)

a) Use Desmos to generate a list of the first 10 partial sums. (Attach a screen shot of the results).
b) Estimate the sum, correct to 4 decimal places
c) How many terms of the sequence of partial sums did your group need in order to make an estimate?

https://www.desmos.com/calculator/bm5ygqipv5

a) $\sum_{n=1}^{\infty} \frac{1}{ne^n}$

b) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3}$

$f(x) = \left(\frac{1}{xe^x}\right)$

x_1	$f(x_1)$	$\sum_{j=1}^{x_1} f(j)$	
1	0.36787944	0.36787944	
2	0.067667642	0.43554708	
3	0.016595689	0.45214277	
4	0.0045789097	0.45672168	
5	0.0013475894	0.45806927	
6	4.131254×10^{-4}	0.4584824	$\approx .4585$
7	1.302689×10^{-4}	0.45861267	$\approx .4586$
8	4.193283×10^{-5}	0.4586546	$\approx .4587$ 4th decimal no longer changing
9	1.37122×10^{-5}	0.45866831	$\approx .4587$
10	4.539993×10^{-6}	0.45867285	$S \approx .4587$ 9 terms
11	1.518336×10^{-6}	0.45867437	
12	5.120177×10^{-7}	0.45867488	
13	1.738715×10^{-7}	0.45867505	
14	5.939491×10^{-8}	0.45867511	

$f(x) = \frac{(-1)^{(x-1)}}{(x)^3}$

x_1	$f(x_1)$	$\sum_{j=1}^{x_1} f(j)$	
1	1	1	
2	−0.125	0.875	
3	0.037037037	0.91203704	
4	−0.015625	0.89641204	
5	0.008	0.90441204	
6	−0.0046296296	0.89978241	
7	0.0029154519	0.90269786	
8	−0.001953125	0.90074473	
9	0.0013717421	0.90211648	
10	−0.001	0.90111648	
11	7.513148×10^{-4}	0.90186779	
12	-5.787037×10^{-4}	0.90128909	
13	4.551661×10^{-4}	0.90174425	$\approx .9017$
14	-3.644315×10^{-4}	0.90137982	$\approx .9013$

4th decimal place still changing at 14 terms... Keep going

35	$2.332362 \times 10^{-[illegible]}$	0.90155384	
36	-2.143347×10^{-5}	0.90153241	$\approx .9015$
37	1.974217×10^{-5}	0.90155215	$\approx .9016$
38	-1.822423×10^{-5}	0.90153392	so 4th d.p
39	1.685801×10^{-5}	0.90155078	still changing.
40	-1.5625×10^{-5}	0.90153516	Need more terms
41	1.450937×10^{-5}	0.90154967	stays $\approx .9015$
42	-1.349746×10^{-5}	0.90153617	Took 41 terms
43	1.257751×10^{-5}	0.90154875	Converges more
44	-1.173929×10^{-5}	0.90153701	slowly than (a)
45	1.097394×10^{-5}	0.90154798	
46	-1.027369×10^{-5}	0.90153771	
47	9.631777×10^{-6}	0.90154734	
48	-9.042245×10^{-6}	0.9015383	
49	8.49986×10^{-6}	0.9015468	

3) For each of the following, (12 points)

a) Does the series converge? Why/Why not?

b) If so, what is the exact sum?

a) $\sum_{n=1}^{\infty} \frac{n^2-3n}{6n^2-5n+9}$ $\qquad \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n^2-3n}{6n^2-5n+9} = \frac{1}{6}$

Since $\lim_{n\to\infty} a_n \neq 0$, the series $\sum_{n=1}^{\infty} \frac{n^2-3n}{6n^2-5n+9}$ diverges by the Test for Divergence

b) $\sum_{n=1}^{\infty} \frac{4^{n-1}}{5^n} = \sum_{n=1}^{\infty} \frac{1}{5}\left(\frac{4}{5}\right)^{n-1}$ geometric with $a=\frac{1}{5}$, $r=\frac{4}{5}$

Since $|r| = \frac{4}{5} < 1$, $\sum_{n=1}^{\infty} \frac{4^{n-1}}{5^n}$ converges and the sum $= S = \frac{a}{1-r} = \frac{1/5}{1-1/5} = 1$

c) $\sum_{n=1}^{\infty} \frac{2}{n(n+2)}$

$= \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+2}\right)$

(Using partial fractions, $\frac{2}{n(n+2)} = \frac{1}{n} - \frac{1}{n+2}$)

So the nth partial sum is

$$S_n = \left(1 - \cancel{\tfrac{1}{3}}\right) + \left(\tfrac{1}{2} - \tfrac{1}{4}\right) + \left(\cancel{\tfrac{1}{3}} - \tfrac{1}{5}\right) + \left(\tfrac{1}{4} - \tfrac{1}{6}\right) + \left(\tfrac{1}{5} - \tfrac{1}{7}\right) + \cdots + \left(\tfrac{1}{n-1} - \tfrac{1}{n+1}\right) + \left(\tfrac{1}{n} - \tfrac{1}{n+2}\right)$$

Anything subtracted gets added 2 terms later

would cancel from 2 terms prior

Telescoping Series, S_n collapses.

$$S_n = 1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2}$$

$\lim_{n\to\infty} S_n = \frac{3}{2}$

$\boxed{\text{series converges } S = 3/2}$

note: should show both of these in S_n